COVID-19 RECESS PAPERS

S6 PURE MATHEMATICS WEEK3

SECTION A

1. Find x if $log_x^8 - log_{x^3}^{64} = 1$
2. Given that $A = tan^{-1}\left(\frac{1}{5}\right)$, $B = tan^{-1}\left(\frac{1}{3}\right)$.
   Prove that $2A + B = tan^{-1}\left(\frac{27}{31}\right)$
3. Differentiate $\frac{\sqrt[3]{2x-1}}{\sqrt{x^2-2}}$ with respect to x
4. Find a vector that is perpendicular to the two vectors $\boldsymbol{a} = \boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$ and $\boldsymbol{b} = -\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$.
5. A polynomial has remainders 3 and -5 when divided by $x - 2$ and $x + 3$ respectively. Find the remainder when the polynomial is divided by $x^2 + x - 6$.
6. A spherical balloon is being filled with air at a rate of $54\pi cm^3 s^{-1}$. Find the rate at which the radius of the balloon increases when the volume is $36\pi\ cm^3$.
7. The tangent to the parabola $y^2 = 8x$ at a point $M(2t^2, 4t)$ is perpendicular to the line whose equation is $3y + x - 6 = 0$. Determine the coordinates of M.
8. Evaluate $\int_0^{0.5} 3sin\theta\ cos3\theta\ d\theta$

SECTION B

9. A curve is given by the equation $y = \frac{4x-3}{x^2-4x+3}$ Find the
   (a) (i) intercepts of the curve (ii) asymptotes of the curve
   (iii) range of values of y within which the curve lies and hence find the turning points
   (b) Sketch the curve
10. (**a**) Find the binomial expansion of $\frac{1}{\sqrt[4]{(1-3x)}}$ in ascending powers of x up to the term in x^3. Hence evaluate $\frac{1}{\sqrt[4]{13}}$ correct to 4 decimal places.

    (**b**) Using the substitution $p = x + \frac{1}{x}$
    solve the equation $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$

11. The points $A(1,\ 2, -3)$, $B(-1,\ 8,\ 0)$, $C(-17,\ 15,\ 8)$ are in the same plane.
    a. Prove that A, B, C are vertices of a triangle

    b. If point P is the foot of the perpendicular from C to $\boldsymbol{AB}$ produced.
    Find the coordinates of P.

12. (a) Find the area bounded by the curve $y = (x-2)^3$ and the lines

(i) $y = 1, \quad y = 8 \quad and \quad x = 0$
(ii) $y = 0, \quad x = 4$ and the y −axis

(b) If the area bounded by the curve in (a) and the lines $x = 2, \quad x = 4 \; and \; y = 0$ is rotated through one revolution about the x −axis, find the volume of the solid generated .

13. Given that $y = tan^{-1}\sqrt{1-x}$

a. Show that

(i) $(2-x)\frac{dy}{dx} + \frac{1}{2}(1-x)^{\frac{-1}{2}} = 0$

(ii) $(2-x)\frac{d^2y}{dx^2} - \frac{dy}{dx} + \frac{1}{4}(1-x)^{\frac{-3}{2}} = 0$

b. Hence determine the Maclaurin's expansion of y up to the term in x^3

14. The points $A(1,\ 2, -3),\ B(-1,\ 8,\ 0),\ C(-17,\ 15,\ 8)$ are in the same plane.

a. Prove that $A,\ B,\ C$ are vertices of a triangle

b. If point P is the foot of the perpendicular from C to $\boldsymbol{AB}$ produced.
Find the coordinates of P.

END